

Let R be an integrally closed domain with quotient field K and θ be an element of an integral domain containing R with θ integral over R . Let $F(x)$ be the minimal polynomial of θ over K and \mathfrak{p} be a maximal ideal of R . Kummer proved that if $R[\theta]$ is an integrally closed domain, then the maximal ideals of $R[\theta]$ which lie over \mathfrak{p} can be explicitly determined from the irreducible factors of $F(x)$ modulo \mathfrak{p} . In 1878, Dedekind gave a criterion to be satisfied by $F(x)$ for $R[\theta]$ to be integrally closed in case R is the localization $Z_{(\mathfrak{p})}$ of Z at the nonzero prime ideal \mathfrak{p} of Z . In 2006, Ershov extended Dedekind Criterion replacing $Z_{(\mathfrak{p})}$ by the valuation ring of any Krull valuation. Using Generalized Dedekind Criterion in this thesis, we have given explicit necessary and sufficient conditions involving only a, b, m, n for $R[\theta]$ to be integrally closed when θ is a root of an irreducible trinomial $F(x) = x^n + ax^m + b$ belonging to $R[x]$, R being a valuation ring. As an application, we have deduced that if K_1, K_2 are algebraic number fields which are linearly disjoint over the field of rational numbers and one of them is a quadratic field with the compositum $A_{K_1} A_{K_2}$ integrally closed, A_{K_i} being the ring of algebraic integers of K_i , then the discriminants of K_1, K_2 are coprime. In an attempt to extend the above result to any pair of algebraic number fields linearly disjoint over $K_1 \cap K_2$, we have proved a more general result which deals with the compositum of integral closures of a given valuation ring R in a pair of finite separable extensions of the quotient field K of R which are linearly disjoint over K . In the course of its proof, we have established an analogue for finite extensions of valued fields of the classical result that the discriminant of an extension of algebraic number fields can be expressed as a product of local discriminants as well as a generalization of the weak Approximation Theorem. We have also generalized an extended version of the classical theorem of factorization of Ore for polynomials with coefficients in henselian valued fields of arbitrary rank