Discriminant whose notion is due to R. Dedekind, is a basic invariant associated to an algebraic number field. Its computation is one of the most important problems in algebraic number theory. For an algebraic number field $K = \mathbb{Q}(\theta)$ with $\theta$ in the ring $A_K$ of algebraic integers of $K$ having $F(x)$ as its minimal polynomial over the field $\mathbb{Q}$ of rational numbers, the discriminant $d_K$ of $K$ and the discriminant of the polynomial $F(x)$ are related by the formula

$$\mathrm{discr}(F) = [A_K : \mathbb{Z}[\theta]]^2 d_K.$$

So computation of $d_K$ is closely connected with that of the index of $\mathbb{Z}[\theta]$ in $A_K$. We characterize those primes which divide the discriminant of $F(x)$ but do not divide $[A_K : \mathbb{Z}[\theta]]$ when $\theta$ is a root of an irreducible trinomial $F(x) = x^n + ax^m + b$ belonging to $\mathbb{Z}[x]$. Such primes $p$ are important for explicitly determining the decomposition of $pA_K$ into a product of prime ideals of $A_K$ in view of the well known Dedekind theorem. As an immediate consequence, we obtain some necessary and sufficient conditions involving only $a, b, m, n$ for $\{1, \theta, \cdots, \theta^{n-1}\}$ to be an integral basis of $K$.
Discriminant is also a valuable tool to find an integral basis of an algebraic number field $K$. The problem of its computation specially for pure number fields has attracted the attention of many mathematicians. We give an explicit formula for the discriminant of squarefree degree pure number fields $\mathbb{Q}(\sqrt[m]{a})$, with $x^m - a$ irreducible over $\mathbb{Z}$, involving only the primes dividing $m$ and the prime powers dividing $a$. In case $K = \mathbb{Q}(\sqrt[n]{a})$ is an extension of degree $n$ of the field $\mathbb{Q}$ of rational numbers, where the integer $a$ is such that for each prime $p$ dividing $n$ either $p\ a$ or the highest power of $p$ dividing $a$ is coprime to $p$, we give a formula for the discriminant of $K$ involving only the prime powers dividing $a$, $n$ and describe explicitly an integral basis of $K$. This clearly takes care of all pure fields $K = \mathbb{Q}(\sqrt[n]{a})$, where either $a$, $n$ are coprime or $a$ is squarefree. Although an integral basis (which yields a formula for the discriminant) of $\mathbb{Q}(\sqrt[n]{a})$ with $a$, $n$ coprime is described in [Proc. Japan Acad. 58A (1982) 219-222], we give counter examples to show that this formula is incorrect.