The central theme is Artin's braid group, and the many ways that the notion of a braid has proved to be important in low-dimensional topology. It will be assumed that the reader is familiar with the very basic ideas of homotopy theory such as the ideas of homotopy equivalence, homomorphism, deformation retractions and the notions of fundamental groups(and its computation) etc. Chapter 1, as a preliminary develop the tools to be used in chapter 2 and 3 of the thesis. The materials here are based on my understanding from the texts: `Algebraic Topology' by Allan Hatcher; `Combinatorial Group Theory' by Magnus, Karrass and Solitar; `Homotopy Theory' by Sze-Tsen Hu. Chapter 2 starts with definition of braid group and deals with the concepts of a braid regarded as a group of motions of points in a manifold. Many algebraic and structural properties of the braid groups of two manifolds are studied, and defining relations are derived for the braid groups of E2 and S2. The materials presented in this section is based on my understanding of Chapter 1, from the text `Braids, Links and Mapping Class Groups' by J.S. Birman [1]. The proof of the theorem 13 is based on my understanding of the paper `Basic Results on Braids', 2004 by J. Gonzalez Meneses [22]. In Chapter 3, we will give some connections between braid groups and mapping class group of the surfaces. Also we compute the mapping class group of the npunctured sphere. The contained of this chapter is based on my understanding of section 4.1 and 4.2 of chapter 4 from the text `Braids, Links and Mapping Class groups' by Birman [1]. The proof of Lemma 7, is based on my understanding from the text `A primer on Mapping class group' by 'Benson Farb and Dan Margalit'. Some of the figures are taken from Birman's book [1] and some other from the Internet. I tried my best to give detailed explanations for each theorems and results which were not that vivid in the original manuscript of Birman [1]. I mentioned the references whenever required in the `Bibliography'.