Abstract

The Inverse Galois Problem over Q(t) is concerned with determining whether a given finite group G occurs as Galois group of some finite regular (ramified) extension, say E of Q(t). Classical Inverse Galois Problem is concerned with solving the above problem over Q instead of Q(t). In this book, we describe various methods to construct Galois extension of Q(t). Due to theorem of Hilbert, also known as Hilbert's irreducibility theorem, which roughly speaking says that if a group G occurs as Galois group over Q(t), then it also occurs as Galois group over Q. Therefore it is enough to work over Q(t). Working over Q(t) has geometric advantage, as extension of Q(t) corresponds to covering of P1 define over Q. The first part of the book (Chapter 1-4) lays the groundwork. It includes definitions, statement of theorems, important propositions that will be required for understanding rest of the book. Another purpose is to keep this book self contained. For readers, who already have a basic knowledge about these topics, may skip the Part I, and directly start reading part II . Chapter 1 gives a short introduction to covering space and fundamental theorem of Galois theory for covering spaces. Chapter 2 gives a short introduction to basic elements of algebraic geometry. the main goal is to show that there is a correspondence between covering of P1 defined over Q and field extension of Q(t). Chapter 3 gives a concise introduction to algebraic groups. If the field extension is not finite, classical Galois correspondence ceases to exist. In this case, we introduce a topology on Galois group, known as Krull's topology which gives G a structure of an algebraic group. As we will see, in some sense it restores this correspondence. Chapter 4 gives a short introduction to theory of rational function fields. We show that concepts of places, primes and valuations are same. Part II (Chapter 5-7) is the heart of the book. It gives logical foundation to rest of the thesis. In these chapters we develop the main theory. we discuss ideas and methods to construct Galois extension of Q(t). The central result of this part is Basic Rigidity Theorem and the Rigidity Criterion (Chapter 7). This method has been very successfull in realizing finite simple groups as Galois group. In Chapter 6, we describe the strategy proposed by E. Noether in 1918 to attack the problem. In Part III (Chapters 8-11) we apply the ideas/methods developed in part II to various finite groups. In chapter 8, we attack the problem using Noether's Trick. In Chapter 9 and Chapter 11, we apply the theory of rigidity and rationally to realize finite groups as Galois group over Q(t). In chapter 11, we have tried to realize the sporadic simple groups as Galois group over Q(t), using the rigidity method. Since we have made extensive use of GAP and ATLAS, Chapter 10 serves the purpose of giving a short introduction on these topics. Appendix contains a short exposition on Hilbert's irreducibility theorem. We have made a program in python to show that An is (2, 3) generated. Using the theory of modular curves, we present a alternating way to realize the alternating groups as Galois group. This uses the fact that An is (2, 3) generated.