Abstract

Riemannian Geometry is the study of Riemannian manifolds which are, roughly speaking, smooth manifolds where we can measure the lengths of the tangent vectors. This helps us to compute lengths of curves in these spaces and talk about shortest paths etc. Hence, we can do geometry on these spaces. In this expository thesis after introducing some basic notions of differentiable manifolds (Chapter 1) we define Riemannian metrics and show the existence of metrics on arbitrary differentiable manifolds (Chapter 2). Then we introduce connections (Chapter 3) and parallel transport. We incorporate a complete proof of the Levi-Civita's theorem on the existence and uniqueness of symmetric connections compatible with the metric. Then using the connections we define geodesics on Riemannian manifolds (Chapter 4). We discuss exponential maps after that and prove Gauss lemma. The thesis ends with introducing curvature of Riemannian manifolds. We show that the sphere S2 and the hyperbolic plane H2 have constant sectional curvature. An important feature of the thesis is that we discuss many examples to illustrate the concepts. However, no originality is claimed on the part of the author. The results and concepts dealt with in this thesis are quite standard. We have closely followed do Carmo's Riemannian Geometry and Barrett O'neill's Ssemi-Riemannian Geometry all the time