Abstract

The main goal of this thesis is to understand the paper 1 on \Division Algebras of Degree 8 with Involutions" by S. A. Amitsur, J.P. Tignol and L.H. Rowen. To this end we set up the foundations of central simple algebras and explore their properties. We shall discuss the Artin-Wedderburn Theorem, the Skolem-Noether Theorem, and some consequences of the same. Further in, we shall define the Brauer group of a field, and what it means to split a central simple algebra. We shall discuss the existence of Galois splitting fields, and then move on to discuss Brauer Groups of certain fields, concluding with Chevalley's Theorem. For a central simple F-algebra A, the dimension [A : F] is a perfect square, say n2. The number n is called the degree of the central simple F-algebra. A central simple F-algebra is defined to be a quaternion algebra, if n = 2. An involution (of the first kind) of A is an antiautomorphism of degree 2 fixing F. It can be shown that, any central simple algebra with involution has degree 2m for some m. A tensor product of quaternion sublagebras with involutions results in a central simple algebra of degree 2m, with the natural involution. Conversely, if a central simple F-algebra with an involution has degree 2m for some m, can it always be written as a tensor product of quaternion F-algebras? We set up the necessary and sufficient conditions for a central simple F-algebra to have involutions, and to be tensor products of quaternion algebras. We use these conditions on \generic abelian crossed products" to construct a counterexample; a division algebra of degree 8 with involution, which cannot be expressed as the tensor product of quaternion subalgebras