Abstract

This work consists of four chapters. In the first three chapters work is done only over field k. The initial part includes the study of division algorithm in the polynomial ring in one variable y over the field. It is shown that ideal description problem (IDP) and ideal membership problem (IMP) are solvable in k[y]. In division algorithm in k[y] for every division one get a unique remainder. But the uniqueness of remainder fails for the division algorithm in the polynomial ring in multiple variables y1; ::::; yn over k. The division algorithm in k[y] helps in solving the IMP. In k[y], r = 0 is the only condition for solving IMP. But because uniqueness of r fails in k[y1; ::::; yn], r = 0 is the sufficient condition for IMP in k[x1; ::::; xn], not the necessary. So some ”good” generators with special properties are needed such that when some polynomial get divided by these generates one get unique remainder and r = 0 mean that polynomial belongs to the ideal. These ”good” generators are called ”Groebner Basis”. So in this work, the ideal membership problem and ideal description problem are solved using Groebner Basis in an algorithmic fashion. Groebner bases are constructed using S - polynomials by Buchberger’s algorithm in this thesis.(see [DO07]) In the third chapter, it is shown that how algebra is linked to geometry. Then the concept of variety is introduced. The ideal-variety correspondence is proved. Ideal and variety connect the algebra with geometry. Varieties provide a geometrical view to the algebraic understanding given by ideals. Then weak, strong and Hilbert’s Nullstellensatz is given, which establishes some connection between ideals and varieties. Next, it is given that any property of varieties leads to some property for ideals in an almost opposite way and vice versa. Construction of radical of an ideal using Groebner basis is given. The decomposition of a variety into irreducibles is given and since there is a correspondence between ideals and variety, decomposition of ideals is also possible. In the last chapter, the work is done on the commutative Noetherian ring with identity. It’s given that for zero-dimensional ideals Groebner basis possesses some special properties. It is possible to recognize a zero-dimensional ideal just by looking at it’s Groebner basis. Then the ways by which one can compute Groebner bases for some basic operations on ideals are given. An algorithm is given which is helpful in checking whether a given ideal is prime or not. Then primary decomposition of zero-dimensional ideals is being presented. One is the general standard way. The second is when the coefficient ring is the field of characteristic 0, then one first makes the ideal in the general position then decompose it. Then an algorithm to primary decompose a general ideal where the ring is a polynomial ideal domain is given. It’s given how one can reduce the high dimensional ideals into zero-dimensional ideals by using the localization at principal prime ideals. So first these general ideals can be turned into some zero-dimensional ideals and then the primary decomposition can be done.