

Abstract

Spectral sequences are a very powerful computational tool in Homological Algebra and Algebraic Topology. They package information about relations between homology groups. The aim of this exposition is to understand their construction and applications in certain contexts. In Chapter II we shall explicitly construct the Spectral Sequence associated with a filtered differential module. This discussion is based on Chapter XV of [2]. It has the advantage of being elementary and thus helping a novice get started. The Exact Couples of Massey, originally introduced in topology, form a broader source of Spectral Sequences. We discuss them in Chapter III, following Chapter VIII of [1]. Chapter IV explains how some of the familiar situations (Filtered differential module, Filtered Chain complex etc.) give rise to exact couples and thereby Spectral Sequences. We shall briefly discuss the question of convergence of spectral sequence in Chapter V. But an explicit discussion will be limited to spectral sequences associated with filtered chain complexes. Chapter VI discusses how double complexes give rise to two different spectral sequences. Then we discuss some applications of spectral sequences to give conceptual proofs of results proved by diagram-chasing in Homological Algebra. The sixth Chapter introduces the Grothendieck spectral sequence. The following is a schematic representation of, how the major topics of this exposition is organized between chapters One may read Chapter II independent of the rest. We believe that from a practical point of view Exact Couples is the most efficient set-up for theoretical constructions of spectral sequences. So in various expositions, we have preferred to use them and total-degree in the construction instead of Filtered differential modules and complementary degree. We have used complementary degree when it shows up naturally such as in the Total complex of a bi-

complex. In the last few chapters, we give external applications of spectral sequences to Topology. We construct spectral sequences arising in non-abelian categories like that of groups, simplicial sets and topological spaces. To this end, in Chapter VIII, we give the most essential introduction to simplicial sets. The focus audience of this thesis is beginners in Homological Algebra. Those who are familiar with the subject may find this exposition rather lengthy. We request them to read diagonally.