Abstract

In this thesis, we will address the problem of finding closed form solutions of a second order linear homogeneous differntial equation. The content of this thesis is based on the paper by Jerald J. Kovacic[?]. In that paper, Kovacic develops an algorithm to determine whether or not a given second order linear homogeneous differntial equation defined over C(x), the field of rational functions in one variable x defined over the field of complex numbers, admits two linearly independent closed form solutions. The algorithm is implemented successfully in computer algebra systems and presently available in MAPLE and MACSYMA. The rest of the thesis is arranged as follows. In chapter 2, we provide basic definitions and terminologies from differntial algebra and from the Galois theory of linear differntial equations. Then, we reduce the problem of finding closed form solutions of second order homogeneous linear differntial equations to the problem of finding such solutions for equations of the kind y00 = ry, where r 2 C(x). The latter has the added advantage that its differntial Galois group can be identified with an algebraic subgroup of SL(2; C). In chapter 3 we prove the Lie-Kolchin Theorem and classify the algebraic subgroups (up to conjugation) of SL(2; C) into 4 distinct classes. In chapter 4, we use the classiffcation of the Galois group of the differntial equation y00 = ry, where r 2 C(x), and obtain conditions that the poles of r must satisfy. In Chapter 5, we study the algorithm in detail and in Chapter 6 we provide several examples to illustrate how the algorithm works. In Chapter 7, we study the proof of correctness of the algorithm.