Abstract

In this thesis, we try to analyze self adjoint operators on a Hilbert space H. This thesis talks about the spectrum, the spectral decomposition and the perturbation of self adjoint operators. The need to study perturbation comes from the setting of Quantum mechanics. If we consider the Hilbert space H = L2(R), then the elements of H are the states of the system. Each observable is represented by a self adjoint linear operator acting on the state space. Each eigenstate of an observable corresponds to an eigenvector of the operator, and the associated eigenvalue corresponds to the value of the observable in that eigenstate. If the operator's spectrum is discrete, the expectation of observables can attain only those discrete eigenvalues. We denote the Hamiltonian by H = ▯ + V where is the Laplacian and V is the potential operator. In the later part of the thesis, we start the theory perturbation in different instances. First we see that the essential spectrum of a bounded operator is invariant under perturbation by a compact operator. Then we see that a small relatively bounded symmetric operator when added to a self adjoint operator gives us a self adjoint operator. Towards the end, we study a special case of rank one perturbations of self adjoint operator. The key result says that the absolutely continuous part of the spectrum stays invariant.