Abstract

In my thesis I have read and understood Professor Michael Singer's proof of the Fano's theorem on linear homogeneous differential equations. In his paper \Algebraic Relations Among Solu- tions of Linear Differential Equations: Fano's Theorem" (see [12]), Singer proves the following theorem: For n <= 6 if there are n- linearly independent solutions y1; ... ; yn of a linear differ- ential equation of order n, over a differential field with an algebraically closed field of constants C, satisfying a non zero homogeneous polynomial over C then all the solutions of the differ- ential equation can be expressed as an algebraic combination of solutions of linear differential equations of order less than n. Furthermore, he proves the existence of differential equations of order n for each n >= 7 such that: 1. there are n linear independent solutions satisfying the relation y2 1 +...+y2n = 0 and that 2. there is a solution which cannot be obtained using algebraic combinations of solutions of linear homogeneous differential equations of lower order than n. This theorem of Singer (see chapter 5) is the meat of this thesis. For the reader's convenience, I have included considerable amount of Picard-Vessiot theory (see chapters 1, 2 and 4) and basic theorems from the theory of algebraic groups needed (see chapters 3 and 6) to understand this theorem of Singer.