Abstract

This work consists of two chapters. The initial part includes the study of the solutions of Cauchy's basic equation which are equations of the form $f(x+y) = f(x) + f(y)$. We start by looking at the solution for this equation when the given function has real domain and range. Various regularity and algebraic conditions leading to the linearity of the solution function are discussed in detail starting from continuity and generalizing it to the condition where only the measurability of the function is needed. Concept of almost additive functions are introduced and the existence of a unique additive function which coincides almost everywhere with almost additive function is proved. Stability of the solution of a Cauchy's equation is discussed in detail with the cases including $|f(x+y) - f(x) - f(y)|$ bounded and unbounded. Solution of the additive functions when the domain and range is extended to complex plane is also discussed. Finally the most general solution of Cauchy's basic equation is constructed using the existence of a Hamel basis for R over Q and the existence of a discontinuous solution for Cauchy's equation is shown. Then second chapter covers the study of convex functions. Various properties of convex functions are discussed. Concept of a weaker form of convexity namely mid convexity of function is introduced and sufficient conditions satisfied by the mid convex functions to be convex are discussed starting from continuity and generalizing it to the condition where the function only needs to be measurable. Finally, a more powerful form of convexity which is log convexity is introduced and the properties of such functions are discussed. Basic knowledge of Measure theory, Functional Analysis and Fourier Analysis is assumed for understanding the topics presented in this work. Any of the non-standard results which are being used are carefully stated and proved.