Abstract

In chapter 1 , we first explain \continuous operations" on vector bundles. For example, direct sum, tensor product, duality and inner product. Clutching theorems are an important technicality to provide the description of tangent bundle of a differentiable manifold and vector bundles over spheres. The Hopf bundle is visualized elegantly using basic quaternion algebra and some diagrams. Finally beautiful construction of classifying spaces is explained in this chapter. In chapter 2 , using some \important properties" of locally trivial bundles, we describe bundles in terms of homotopy properties of topological spaces. In chapter 3 , starting with simple notion of symmetrization of an abelian monoid, we define the group K(X) of X using the isomorphism classes of vector bundles over X. To extend the study of the properties of the vector bundles, we need further geometric ideas and constructions which lead to deeper properties of vector bundles. One of them is the Bott periodicity theorem, an important result for calculation of K-theory. In chapter 4 , for each vector bundle, we define \Chern classes" using cohomology ring of classifying spaces (with suitable coefficient ring) in an axiomatic way. By means of these classes, we construct a fundamental homomorphism, the \Chern character" from K(X) to Heven(X;Q). In chapter 5 , we explain Gysin sequence for describing the K-groups of spaces by reducing them to a description in terms of the usual cohomology groups of spaces. Then we prove that the only spheres which admit an H-space structure are S1, S3 and S7 and that these are the only parallelizable spheres.