Abstract

Given a vector bundle, a natural question to ask is whether it is trivial. This is equivalent to the statement that the bundle admits as many nowhere vanishing, linearly independent vector fields as its rank. Hence, the obstruction to triviality is vanishing of some section. We try to understand this question by studying some well-known topological invariants of real and complex vector bundles. We will construct Stiefel- Whitney classes of real vector bundles and Chern classes of complex vector bundles. These invariants are actually cohomology classes in the cohomology ring the base space B and trivial bundles have trivial invariants. In addition, they also help in distinguishing between different bundles over the same base: in that, bundles with different invariants are different. We first study Chern-Weil theory which uses differential geometry to construct de Rham cohomology classes for a differential manifold. We will then study Stiefel-Whitney and Chern classes using algebraic topology for CW complexes.