Abstract

In this thesis we study the sedimentation of particles in a Stokesian fluid, that is, in the limit where viscosity dominates and inertia is ignored. This is a classical n-body problem with long-ranged hydrodynamic interactions which is very difficult to solve. If an analytical form of the interaction between two particles is known, one can do pairwise addition of forces and torques on a particles due to the nearest neighbours and arrive at the discrete form of the equations of motion. But usually it is not at all easy to get the analytical form of interaction by solving the Stokes equation for a particle of general shape. Our interest is to study the collective behavior of anisotropic sedimenting particles. Taking a different approach to this problem we build up a field theory for the displacement and orientation fields of a lattice of sedimenting particles and construct the mobility for the lattice from general symmetry arguments in the continuum limit. We do this for an array of spherical particles (as done by Lahiri and Ramaswamy, PRL 79 1150 (1997)), apolar axisymmetric particles (disks, rods, ellipsoids or any surface of revolution with up-down symmetry) and polar axisymmetric particles (cones, hemispheres or any surface of revolution with up-down asymmetry). We go back and forth from discrete to continuum version of the equations to get maximum knowledge about the interactions between the particles. In this investigation we also do experiments with disks shaped particles and observe various intriguing dynamics of a pair of disks. In chapter 1 we give a brief introduction to the hydrodynamic approach for sedimentation and discuss Crowley instability [1]. In chapter 2 we present the continuum dynamical model for the lattice of sedimenting spherical particles and see its consistency with the hydrodynamic results. This is done by defining a displacement field of the lattice ($\vec{u}$) and writing its equations of motion from general symmetry arguments. Lahiri and Ramaswamy write a dispersion relation which incorporates Crowleys instability as a special case. We then study a more complicated problem by adding an orientation degree of freedom to the sedimenting particle. We observe the dynamics of single disks and pair of disks (see chapter 3) and find periodic behavior for a pair of disk for a large set of initial configurations. A detailed study is needed for this. Once an additional degree of freedom is added to the particles, an obvious question which arises is how the collective behavior of the lattice of particles changes.We find that the orientation degree of freedom competes with clumping and in certain initial configurations can even lead to lattice dilation and orientation waves. In chapter 4 of the thesis we construct a continuum dynamical model for an array of apolar axisymmetric particles like disk, rods etc. by defining the orientation field $\vec{K}$, in addition to the displacement field $\vec{u}$. We construct the equation of motion from symmetry arguments and then find the linear dispersion relation. The equation for the orientation variable tells us that there is no rotation of particles if the gradient of the displacement and orientation field is zero. This is ultimately a consequence of the time-reversal symmetry of the system. For array of disks falling one above the other we find the possibility of orientation waves of the type proposed by Wakiya [2]. At the end of this chapter a consistency of the continuum equations with the hydrodynamic solution can be appreciated. One can relax the $K \rightarrow -K$ symmetry in the system and construct the mobility for an array of polar axisymmetric particles like cones, hemispheres etc. in the continuum limit. We do this in chapter 5 and show the possibility of rotation of cones in a lattice even for the case when the gradient of both the displacement and orientation field is zero. This rotation make the orientation vector asymptotically align with the direction of gravity. A plausible form of this rotation is found just by analyzing the symmetry of the system. All the accounting required for the construction of mobility tensor for various parts of this thesis is given in the Appendices.