Abstract

In this thesis we focus on algebraic curves de ned over an algebraically closed eld of characteristic zero. We begin by giving some basic de nitions of terms in chapter 1 which will be used throughout. In chapter 2 and chapter 3 we de ne singular and normal varieties. We show that the nonsingular varieties are normal. Our main aim in these two chapters is to resolve the singularities of curves. We will show that there exists a normalization of any variety. We will conclude that normalization resolves the singularities of the curve. We then will give the construction of blowup of a surface at a point and show that an embedded curve can be resolved after nitely many blowups of the surface. In chapter 5 and chapter 6 we discuss the notion of Weil divisors and Cartier divisors. In chapter 7 we look at the vector space of rational functions constructed with respect to a given divisor. Given a divisor we will see in chapter 8 that there is 1-1 correspondence between Cartier divisors and invertible sheaves on a projective variety, in particular a nonsingular projective curve. After having developed the necessary machinery we will then prove the Riemann- Roch theorem for curves and look at some of its applications in chapter 10. In the next chapter given a nite morphism between two curves we look at relation between their genus. And nally, we show that any nonsingular, projective curve can be embedded in P3: