Abstract

It is a fundamental problem in Riemannian geometry to try and capture the geometry of a Riemannian manifold by certain of its geometric invariants. In this thesis we consider closed (compact without boundary) Riemannian manifolds M and the action of the geodesic ow gtM on the unit tangent bundle SM. It turns out that if M has negative sectional curvature then the geodesic ow gtM has significant in uence on the geometry of M; for instance, it is a well known fact that a typical geodesic in M is dense. This is in sharp contrast with the case of geodesics on the unit sphere in R3, where every geodesic is a great circle; in particular none of the geodesics is dense. The classification theorem for surfaces says that a closed surface M in R3 is homeomorphic to either a sphere or a torus or a surface of higher genus. The genus of a surface determines its Euler characteristic, which is a topological invariant; more precisely, the Euler characteristic x(M) of a surface M of genus g is 2 - 2g. The celebrated Gauss Bonnet theorem relates the Euler characteristic of a surface M to its Gaussian curvature K by the formula Z M KdA = 2πx(M) where dA is the area form in M. A consequence of the Gauss Bonnet formula is that the sign of curvature on a given closed surface M, if the same sign holds at all points of M, is restricted to a single choice. For example on a sphere S2, whose Euler characteristic is 2, a negative sign on the curvature at all of its point is not possible, whereas such a thing is possible on a surface of genus >= 2. The classical uniformization theorem for surfaces precisely confirms this possibility. That is, a surface M of genus >= 2 admits a metric of constant negative curvature -1. The main theorem discussed in this thesis concerns metrics of non positive curvature on a surface M of genus >= 2 and proves that such metrics are determined up to isometry by the action of the geodesic ow gtM on SM. More precisely, we will discuss a proof of the following theorem. Theorem 0.0.1 (Croke, 1990). Let N be a closed surface of genus >= 2 with non pos- itive sectional curvature and M be a compact surface whose geodesic ow is conjugate to N via F; i.e., F : SM 7! SN is a C1-diffeomorphism such that F o gtM = gtN o F for all t then F = gK N o df, where f is an isometry from M to N and K is a fixed number.