Abstract

The aim of this report is to highlight the major developments in the topics of dimension subgroups and augmentation quotients. The identification of certain normal subgroups determined by the powers of the augmentation ideal of a group ring R(G), known as the dimension subgroups, is one of the most challenging problems in group rings. The study of these subgroups is supposed to be originated by W. Magnus [Mag35] when he conjectured that for any group G, the lower central and the integral dimension series coincide. The works of Magnus [Mag37] and E. Witt [Wit37] implied that the conjecture is true for finitely generated free groups. The conjecture remained undecided until more than three decades later, it was proved to be false by E. Rips [Rip72]. As of now, it is known that the first three terms of the integral dimension series coincide with the lower central series but the same cannot be said for the subsequent terms. The first chapter of this report gives a broad overview of the results concerning dimension subgroups. We begin with some basic definitions and as an example, compute the second dimension subgroup of the group ring R(T), where T = Q/Z. We then state and prove the theorem of Magnus regarding dimension subgroups of free groups using the theory of Lie algebras. We then list some special cases in which we can compute these subgroups. The results concerning integral dimension subgroups in low dimensions are stated next. In this section, we define polynomial maps and give their connection with dimension subgroups, which forms a motivation for the next chapter. The canonical filtration of the augmentation ideal by its powers gives us two sequences of Abelian groups, known as the polynomial groups. These were first computed by I.B.S. Passi for cyclic and elementary Abelian groups [Pas68b]. One of these, known as the sequence of augmentation quotients has been extensively studied by many authors. The second chapter begins with a brief description of some early results. One of the important results regarding augmentation quotients due to F. Bachmann and L. Grunenfelder [BG74] states that for all finite groups, the sequence of augmentation quotients is periodic. In particular, for finite Abelian groups, it is stationary. This stationary structure was given by A. Hales in terms of generators and relations [Hal85]. We study this result in some detail. The structure of augmentation quotients for all finite Abelian groups was described by S. Chang and G. Tang [CT11]. Description of their proof forms the next part of this chapter. Finally, we conclude by summarising some results regarding non-Abelian groups, in particular, the case of symmetric groups [ZT08]. We also list some developments for the non-Abelian case [LL79, ZY09, ZY10].