**Abstract**

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| This is an exposition on the theory of an important class of algebraic curves– elliptic curves. The thesis, comprising of two parts, discusses their theory over some fields of arithmetic interest, viz., finite, local, and number fields. The main results in the first part are– Hasse’s theorem and the criterion of Néron-Ogg-Shafarevich. Hasse’s theorem gives an estimate of the number of F q -rational points on an elliptic curve E defined over a finite field F q , and the criterion of Néron-Ogg-Shafarevich helps characterizing nonsingularity of the reduction of an elliptic curve over a local field. The second part deals with the theory of elliptic curves over a number field. For an elliptic curve E defined over a number field K, it turns out that the abelian group of K-rational points, E(K) is finitely generated. This result, known as the Mordell-Weil theorem is proved in detail using the height functions. As a result, E(K) is completely determined if we can compute its torsion part and find its rank. Nagell-Lutz theorem combined with some local-global arguments make it possible to compute the torsion part, however the problem of computing the rank associated to an elliptic curve is still an unsolved one. Finally, ’the descent by isogeny’ method is discussed with an illustrating example. This helps in computing E(K), in the case when there is a rational 2-torsion point. |

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