

Abstract

We consider the Lie groups $SU(n, 1)$ and $Sp(n, 1)$ that act as isometries of the complex and the quaternionic hyperbolic spaces respectively. We classify pairs of semisimple elements in $Sp(n, 1)$ and $SU(n, 1)$ up to conjugacy. This gives local parametrization of the representations ρ in $\text{Hom}(F^2, G)/G$ such that both $\rho(x)$ and $\rho(y)$ are semisimple elements in G , where $F^2 = \langle x, y \rangle$, $G = Sp(n, 1)$ or $SU(n, 1)$. We use the $PSp(n, 1)$ -configuration space $M(n, i, m - i)$ of ordered m -tuples of distinct points in \mathbb{H}^n , where the first i points in an m -tuple are boundary points, to classify the semisimple pairs. Further, we also classify points on $M(n, i, m - i)$. Particularly interesting coordinates occur for lower values of n . The conjugacy classification of pairs is then applied geometrically to obtain Quaternionic hyperbolic Fenchel-Nielsen type parameters for generic representations of surface groups into $Sp(2, 1)$ and $Sp(1, 1)$.