Synopsis

Nonlinear dynamical systems exhibit many counterintuitive phenomenon and exotic spatiotemporal patterns. On one hand, phenomenon like chaos and stochastic resonance in individual nonlinear dynamical units challenge our everyday intuitions, on the other, complex systems consisting of interacting nonlinear dynamical units provide us a framework to model many physical, biological, social and engineering systems thereby enabling us to get deeper insights into wide ranging complex phenomena. The work in this thesis is divided into two broad categories. First, we explore the application of nonlinear systems in the design of computing devices. Second, we attempt to broaden our understanding of nonlinear systems in general by investigating the emergence of spatiotemporal patterns in complex networks with time-varying connections. Specifically in the first part, we study the possibility of utilizing the phenomenon of stochastic resonance in bistable or multi-stable nonlinear dynamical systems to implement memory and logic function. This phenomenon has commonly been referred to as "Logical Stochastic Resonance (LSR)". We demonstrate how noise enables a bistable system to behave as a memory device, as well as a logic gate for sub−threshold input signals. It is shown how this system can implement memory using noise constructively to store information. Namely, in some optimal range of noise, the system can operate flexibly, both as a AND/OR gate and a Set−Reset latch, on variation of an asymmetrizing bias. Then we examine the intriguing possibility of obtaining dynamical behavior equivalent to LSR in a noise-free bistable system, subjected only to periodic forcing, such as a sinusoidal driving or rectangular pulse trains. We find that such a system, despite having no stochastic influence, also yields phenomenon analogous to LSR, in an appropriate window of frequency and amplitude of the periodic forcing. The results are corroborated by electronic circuit experiments. Next we demonstrate how width of the optimal noise window can be increased by utilizing the constructive interplay of noise and periodic forcing, namely, noise in conjunction with a periodic drive enables the system to yield consistent logic outputs for all noise strengths below a certain threshold. Thus we establish that in scenarios where noise level is below the minimum threshold required for LSR (or stochastic resonance in general), we can add a periodic forcing to obtain the desired effects. We have also ix shown that the periodic forcing results in lower latency effects and reduces the switching time, leading to faster operation of the devices. Further, if a LSR element is coupled to another LSR element with a lower potential barrier, then it is able to adapt to varying noise intensity, so that its operation remains robust even under high noise conditions. Lastly, we test these concepts in vertical-cavity surface-emitting lasers (VCSELs) which are widely used for high-bit-rate data transmission because of their various advantages over conventional edge emitting lasers like low threshold current, single-longitudinalmode operation, higher modulation bandwidth and circular output beam profile. We attempt to enhance the the operational range of VCSEL based stochastic logic gate by adding a periodic signal. The enhancement is observed in form of decrease in the minimum bit time necessary for successful operation or increase of size of the optimal noise window. In the second part of this thesis, we study the behaviour of nonlinear dynamical elements coupled with each other. Firstly, we study the impact of small heterogeneity in signals applied to globally coupled nonlinear bistable elements. In absence of coupling, the collective response is simply the average of response to all the uncorrelated signals. When the elements are coupled and a bias is applied, we find that even a very small number of heterogeneous inputs are able to drag the collective response towards the stable state of the minority inputs. In our explicit demonstration we have taken Schmitt triggers as the nonlinear bistable elements, and the inputs are encoded as voltages applied to them. The average of the output voltages of all Schmitt triggers corresponds to the output of the system. We also observe that the minimum heterogeneity that can be detected scales with the ratio of threshold voltage to the source voltage of the Schmitt triggers, and can be be brought down to the limit of single bit detection. In last two works, we focus on changes in emergent phenomenon when the underlying interaction network is dynamic and the connections evolve with time. Such time variations represent the evolution of interactions over time or any discontinuities in interactions, i.e. when the nodes interact only for limited time. These evolving

interaction patterns are commonly found in social networks, communication, biological systems, spread of epidemics, computer networks, world wide web etc and have been shown to result in significantly different emergent phenomena in complex systems. In the first problem, we study the impact of time varying network topology in epidemic spreading. We study a simple model mimicking disease spreading on a network with dynamically varying connections, and investigate the dynamical consequences of x switching links in the network. Our central observation is that the disease cycles get more synchronized, indicating the onset of epidemics, as the underlying network changes more rapidly. This behavior is found for periodically switched links, as well as links that switch stochastically in time. We find that the influence of changing links is more pronounced in networks where the nodes have lower degree, and the disease cycle has a longer infective stage. Further, in periodically switching links, we observe finer dynamical features, such as beating patterns in the emergent oscillations and resonant enhancement of synchronization, arising from the interplay between the time-scales of the connectivity changes and that of the epidemic outbreaks. In the second problem, we study the stability of the synchronous state in evolving networks. Many earlier studies have analyzed the stability of the synchronous state by linearizing the dynamical equations. But this approach is valid only in case of small perturbations of the synchronous state. In general, the dynamical equations governing the dynamics over the nodes are nonlinear and the higher order terms no longer remain negligible in case of large perturbations. In such cases, the basin stability approach may be used to complement the linear stability analysis. The basin stability paradigm is particularly useful in case of time varying networks as it can be applied to a large class of systems whereas the linear stability analysis can be done only in some specific cases. In our study, we consider synchronization of chaotic R¨ossler oscillators over Watts−Strogatz networks. We vary the fraction of random links, p, to cover broad range of networks varying from regular ring topology for $p = 0$ to random networks for $p = 1$. We find that for sufficiently fast re-wirings, the time varying networks can be approximated by static time averaged networks. Using the basin stability framework, we are able to estimate the rewiring frequency at which the network can be approximated by the static time average. Further we are also able to get insight into how the transition from a static to a time averaged case takes place and show how the stability range changes at different rewiring timescales. We find that not only the basin stability of small world networks highest in static cases as reported earlier, but they approach the time averaged coupling case fastest. Further, we find that faster rewiring networks synchronize quickly and the impact of rewiring is maximum when the number of neighbours is less. Lastly, we show that linear stability analysis alone is not sufficient to accurately predict the stability of synchronized states in time varying networks and the basin stability analysis should also be used to complement the analysis. In the last chapter of the thesis, we conclude our findings and summarize the important results of all the chapters. We also list some possible extensions of the works presented xi in this thesis.