Abstract

Synchronization of complex networks has attracted wide research interest, from fields as diverse as ecology and sociology to power grids and climatology. In this work, we have studied collective dynamics of a group of coupled bi-stable elements. The work in this thesis is broadly divided into three parts. In the first part, we investigated the dynamics of networks of bistable elements with varying degrees of randomness in connections, considering both static random links and time-varying random links. We demonstrated how the presence of a few dissimilar elements affected the collective features of this system, and found that a network with random links is hyper-sensitive to heterogeneity. Namely, counter- intuitively, even a small number of distinct elements manages to drastically influence the collective dynamics of the network, with the mean-field swinging to the steady state of the minority elements. We found that the transition in the collective field gets sharper as the fraction of random links increases, for both static and time-varying links. We also demonstrated that networks where the links are switched more frequently, synchronize faster. Lastly, we showed that as global bias tends to a critical value, even a single different element manages to drag the entire system to the natural stable state of the minority element. So it is evident that when coupling connections are random, a network with even a very small number of links per node, has the ability to become ultra sensitive to heterogeneity.

In the second part, we consider the collective dynamics of a group of bistable elements connected in different network topologies, ranging from regular rings and small-world networks on one hand, to deterministic scale-free and random scale-free networks on the other. The basic question we address is the following: are there features of the underlying connection network that provide consistent markers for the emergence of complete synchronization and the robustness of the synchronized state? We focus on the correlation between network properties and global synchronization features. Our central result is that, while networks properties can provide indicators of synchronization within a network class, they fail to provide consistent indicators across network classes. This suggests that local properties determined by the connection network does not provide a complete picture of global measures of synchronization. Our observations then have potential applications. For instance, if one needs to achieve synchronization in a network of bi-stable elements, such as electronic circuits, constrained by certain connection properties, our analysis can guide the choice of preferred topology. More importantly, in the context of the general understanding of dynamical networks, our observations suggest important caveats to correlating network features to global dynamical phenomena.

In the third part, we attempt to identify the nodal property that most significantly influences the global stability of the network. We explore bistable elements, connected in different network topologies, ranging from regular rings to random scale-free and star networks. We focus on the response of this network to localized perturbations on a sub-set of nodes. The central question we investigate here is the following: what characteristics of the nodes (if any) significantly affect the global stability?. We consider three properties of the nodes: degree, betweenness centrality and closeness centrality and study the influence of propagation of perturbations emanating from these nodes. We show that perturbing nodes with high closeness and betweenness-centrality significantly reduces the capacity of the system to return to the desired stable state. This effect is very pronounced for a star network which has one hub node with significantly different closeness/betweenness-centrality than the peripheral nodes. The comparison between the global stability of these two classes of networks, namely Random Scale-Free Networks with m=1 and m=2, provides clear indications that the betweenness centrality of the perturbed node is more crucial for dynamical robustness, than closeness centrality or degree of the node. This result is important in deciding which nodes to safeguard in order to maintain the collective state of this network against targeted localized attacks.