Integration in Finite Terms with Special Functions: Polylogarithmic Integrals, Logarithmic Integrals and Error Functions.

Abstract

The thesis work concerns the problem of integration in finite terms with special functions. The main theorem extends the classical theorem of Liouville in the context of elementary functions to various classes of special functions: error functions, logarithmic integrals, dilogarithmic and trilogarithmic integrals. The results are important since they provide a necessary and sufficient condition for an element of the base field to have an antiderivative in a field extension generated by transcendental elementary functions and special functions. A special case of the theorem simplifies and generalizes Baddoura's theorem for integration in finite terms with dilogarithmic integrals. The main theorem can be naturally generalized to include polylogarithmic integrals and to this end, a conjecture is stated for integration in finite terms with transcendental elementary functions and polylogarithmic integrals.