Title: Pattern formation in Complex Dynamical Networks

Abstract

The work in this thesis is centered around the exploration and characterization of emergent behaviour, especially synchronization and chimera states, in mathematical models of complex systems and networks. We have also focused on the mechanisms that can effectively control complex networks of chaotic systems to steady states and the robustness of these steady states.

In the first research problem, we have studied the dynamics of two coupled nonlinear delay differential system modeling the El Niño Southern Oscillation (ENSO) phenomenon. We have explored the dynamics of ENSO phenomenon in the space of three parameters: self-delay, delay, and inter region coupling strengths. The emergence or suppression of oscillations in our models is a dynamical feature of utmost relevance, as it signals the presence or absence of ENSO-like oscillations. We then investigate the basins of attraction of the different dynamical attractors arising in our model. Mapping of the basins of attractions from our numerical results suggests that instead of the single value criterion, an interval should be used as a criterion to estimate the El Niño or La Niña progress. Thus our dynamical model may help in providing a potential framework to understand patterns in the SST anomalies in different coupled sub-regions and might be useful in the forecasting of El Niño/La Niña years.

In the second problem we have studied star networks of chaotic oscillators, with all end-nodes connected only to the central hub node, under diffusive coupling, conjugate coupling and mean-field type coupling. We observed the existence of chimeras in the end-nodes, which are identical in terms of the coupling environment and dynamical equations. Namely, the symmetry of the end-nodes is broken and co-existing groups with different synchronization features and attractor geometries emerge. Surprisingly, such chimera states are very wide-spread in this network topology, and large parameter regimes of moderate coupling strengths evolve to chimera states from generic random initial conditions. Thus it is evident that star networks provide a promising class of coupled systems, in natural or human-engineered contexts, where chimeras are prevalent.

In the third problem, we have established a mechanism to control intrinsically chaotic meta-population to the steady states and periodic behaviour. For that, we have explored Random Scale-Free networks of populations, modelled by chaotic Ricker maps, connected by transport that is triggered when population density in a patch is more than a critical threshold level. Our central result was that threshold-activated dispersal leads to stable

fixed populations, for a wide range of threshold levels. Further, suppression of chaos is facilitated when the threshold-activated migration is more rapid than the intrinsic population dynamics of a patch. Additionally, networks with a large number of nodes open to the environment, readily yield stable steady states. We have also demonstrated that in networks with very few open nodes, the degree and betweeness centrality of the node open to the environment has a pronounced influence on control.

In the last problem, we have investigated the collective dynamics of multi-stable chaotic systems connected in different network topologies, ranging from rings and small-world networks to scale-free networks and stars. We estimate the dynamical robustness of such networks by introducing a variant of the concept of multi-node basin stability, which allows us to gauge the global stability of the dynamics of the network in response to local perturbations affecting a certain class of nodes of a system. We show that perturbing nodes with high closeness and betweeness-centrality significantly reduces the capacity of the system to return to the desired state.