Abstract

Hermann Weyl, in his famous book *'The Classical, their Invariants and Representations'*, coined the phrases "classical groups" to describe certain families of groups of linear transformations. Even years after its introduction, these groups continue to retain their importance in mathematics (especially in the subject of linear Lie groups) and has applications in both classical and modern physics.

We begin the thesis by introducing classical groups and study them from the point of view of linear algebraic groups. We then develop the theory in order to obtain basic results on their representations and lie algebras. We further move on to study the structure of classical groups. We show that for a classical group G, the subgroup of diagonal elements H is the maximal torus and every maximal torus is conjugate to H. We find the decomposition of $\mathfrak{g}$, the lie algebra of classical group G in terms of root and root spaces, under the adjoint action of H.

In the second part of the thesis, we look at the 'tensor product problem'. To put it into perspective, consider a finite dimensional simple lie algebra $\mathfrak{g}$. It is known that if W is a tensor product of finite dimensional irreducible modules of $\mathfrak{g}$, then W is completely reducible. The major goal of tensor product problem is to determine the irreducible $\mathfrak{g}$-modules of W along with their multiplicities. We look at one of the recent studies aimed at tackling this problem.