

Abstract

Theta functions are special functions in several complex variables. Theory of theta functions holds significance in many areas of mathematics including number theory, algebraic geometry, among others. Most common form of theta functions appear in theories of elliptic curves and elliptic functions.

During my dissertation project, my aim was to study the Riemann theta function $\theta(z, t)$ for $z \in \mathbb{C}$, $t \in \mathbb{H}$, and analyzing its behaviour with respect to variable z . Restricting the variables to reals, we also realize the function as a fundamental periodic solution to heat equation when x lies on a circle (S^1). Apart from the standard theta function, we can construct its variants also (with rational characteristics). Constructively, using them, we try to show how these functions can be used to embed the torus ($\mathbb{C}/\mathbb{Z} + \mathbb{Z}t$) inside a complex projective space and in particular, take the case of projective 3-space (\mathbb{P}^3). Then we show using theta relations, how the equation for the image curve can be found and in the process, we obtain many Riemann theta relations.

Next, I try to define doubly periodic meromorphic functions (elliptic functions) on the elliptic curve E_t using the variants of theta functions.