Bell nonlocality of quantum theory refers to the nonclassical correlations obtained by local measurements on spatially separated entangled subsystems. Bell nonlocality is a resource for device-independent quantum information processing. Quantum discord was introduced as a measure of quantum correlations which captures nonclassical correlations in separable states as well. Recently, it has been shown that non-null quantum discord is a resource for quantum information processing. Quantum correlations forms a subset of the set of nonsignaling boxes. This allows us to characterize quantum correlations as a convex combination of the extremal boxes of the nonsignaling polytope which are Popescu-Rohrlich boxes (maximally nonlocal boxes) and local deterministic boxes. There exists multiple decomposition of quantum correlations in the context of the nonsignaling polytope. I find that the existence of Popescu-Rohrlich box decomposition for local boxes associates two notions of discord which capture nonclassicality of quantum correlations originating from Bell nonlocality and EPR-steering. I introduce, Bell and Mermin discord, and show that any bipartite nonsignaling box admits a three-way decomposition. This decomposition allows us to isolate the origin of nonclassicality into three disjoint sources: a Popescu-Rohrlich box, a maximally EPR-steerable box, and a classical correlation. Interestingly, I show that all non-null quantum discord states which are neither classical-quantum states nor quantum-classical states can give rise to nonclassical correlations which have non-null Bell and/or Mermin discord for suitable incompatible measurements. I introduce two notions of genuine discord, which are the generalizations of Bell and Mermin discord to the multipartite scenario, to characterize the presence of genuine nonclassicality in quantum correlations.