The work in this thesis broadly falls in the domain of interdisciplinary research on the dynamical character of complex systems. It includes mathematical modeling, numerical analysis, network theory and ideas from the theory of dynamical systems, to characterize the rich variety of patterns manifested by such systems. In particular, we have focused on the effect of the interplay between structural complexity (which reflects the topology of connections) and inherent dynamical complexity arising from the nonlinearity of the local dynamics on the emergent spatiotemporal behavior of such systems. First problem of the thesis deals with the study of collective dynamics of coupled limit cycle oscillators with nearest neighbor nonlinear interactions. To our surprise, we found that the system undergoes an explosive growth and becomes unbounded as coupling strength becomes greater than a critical value. Further, we found that rewiring such a system from a regular ring topology to a random network, with time-varying switching of connections, leads to suppression of the unbounded growth. We then investigated the bounds on the time scale of network switching which would guarantee the existence of bound state as well as for synchronized state. We first tried to understand the occurrence of explosive growth through linear stability analysis but later found that that it is a weak measure and carried out further analysis with a global measure (basin stability) which effectively captured the bounded-unbounded transition. This problem is significant from both theoretical and applied point of view. From theoretical perspective, the work poses further questions about finding the origin of growth of instabilities in such highdimensional systems and also its relation to the type of interactions between the subsystems. Also from the stand point of potential applications, this work suggests a method to control and prevent catastrophes in coupled oscillators that are commonplace in a variety of engineered systems. In the second research problem, we studied the influence of local noise on a generalized network of populations having positive and negative feedbacks. The population dynamics at the nodes is well mixed and modeled by a nonlinear map (Ricker map), typically chaotic, and allows cessation of activity if the population falls below a threshold value (Allee effect). We then investigated the global stability of this large interactive system, as indicated by the average number of nodal populations that manage to remain active. Our central result suggests that the probability of obtaining active nodes in this network is significantly enhanced under fluctuations. Further, we found a sharp transition in the number of active nodes as noise strength is varied, along with clearly evident scaling behaviour near the critical noise strength. From ecological perspective, this study addresses the classic problem of ecosystem's stability from a different perspective as the method used here is non-perturbative in nature as we have applied global measures to gauge the stability of ecosystem in presence of large perturbations, rather than usual local stability analysis in response to very small perturbations. Moreover, much of the effort in theoretical ecology has been devoted to mechanisms that promote stable co-existence of species at equilibrium. In this context, our results suggests the possible phenomena of species coexistence in an open variable environment with an emergent non-equilibrium steady state having large number of active species. In some sense, second problem inspired us to pursue another fascinating problem in theoretical ecology which is to understand the origin of different dynamical behavior arising purely out of interactions in a multi-species community. Most studies from the literature focused on the \diversity-stability" issue and totally ignored the connection with dynamical behavior of the system. Our model has twofold benefits. First, it explains the emergent complex dynamics exhibited by real multispecies communities as evident from the empirical data, by modeling the interactions and classifying the different dynamical behavior as a function of some suitably chosen interaction parameter. Secondly, it also helps us in understanding the so called \diversity-stability" issue from dynamical view point as opposed to popular choice of looking at the eigenvalues of the interaction matrix only. Our main observation is that the population density, reflecting the biomass yield, displays distinct non-monotonic scaling behaviour with respect to the product C (where is net interaction strength and C tells us the fraction of species interacting with other species), implying that survival is dependent not merely on the number of links, but rather on the combination of the sparseness of the connectivity matrix and the net interaction strength. Interestingly, in an intermediate window of positive C, the total population density is maximal, indicating that too little or too much positive interactions is detrimental to survival. Further, at the local level we observed marked qualitative changes in dynamical patterns, ranging from anti-phase clusters of period 2 cycles and chaotic bands, to fixed points, under the variation of mean of the interaction strengths which finally tells us what kind of dynamics is most optimal for resource consumption and consequently the maximal biomass production. The last problem, is on the study of f-node basin stability of the synchronized state on various network topologies with chaotic Rössler oscillators on the nodes. Basically, here we are probing the effect of spatially localized perturbations on the global stability of the synchronized state of the system. So far, no generic framework exists for spatially localized perturbations and we are hoping that our approach would be a first step in this direction. The concept of f-node basin stability is particularly useful in network topologies with heterogeneous degree distributions like deterministic scale free networks, bipartite networks, etc. Therefore, we are attempting to measure and compare the basin stability of synchronized state where perturbation is applied to nodes having a particular degree. The results so far, have led us to an important observation which says that as the network becomes more ordered/structured, its ability to sustain synchronized state decreases. In summary, we have explored a broad range of problems concerning the influence of network topology and complexity of the dynamics on the collective behavior of the system. Specifically, in one problem we explored the effectiveness of time-varying interactions in suppressing explosive instabilities(blow-ups) occurring in the system. In another problem we explored the statistical aspects of the constructive role of noise in enhancing species activity in a complex multi-species ecosystem with Lotka-Volterra type interactions between the species. Further, in yet another problem relevant to theoretical ecology, we explored the effect of imbalance of interactions in a multi-species community on the dynamics of the species which in turn connects with the survival of the species. Thus, the problems undertaken in this thesis have yielded results that have enhanced our current understanding of complex dynamical networks from a theoretical, as well as an applied, perspective.